Low Impedance Parallel Wire Transmission Lines
Originally in VHFer in 1966 but still applicable
by
Dr. Gerald N. Johnson, KØCQ

Back about 1964, I struggled to design a low impedance balanced line to match an array.

By 1966, my research showed the common formula was incorrect for low impedance lines.

So wrote this article and submitted it to VHFer.

.

LOW IMPEDANCE PARALLEL TRANSMISSION LINES

by Gerald N. Johnson, KØCQA*

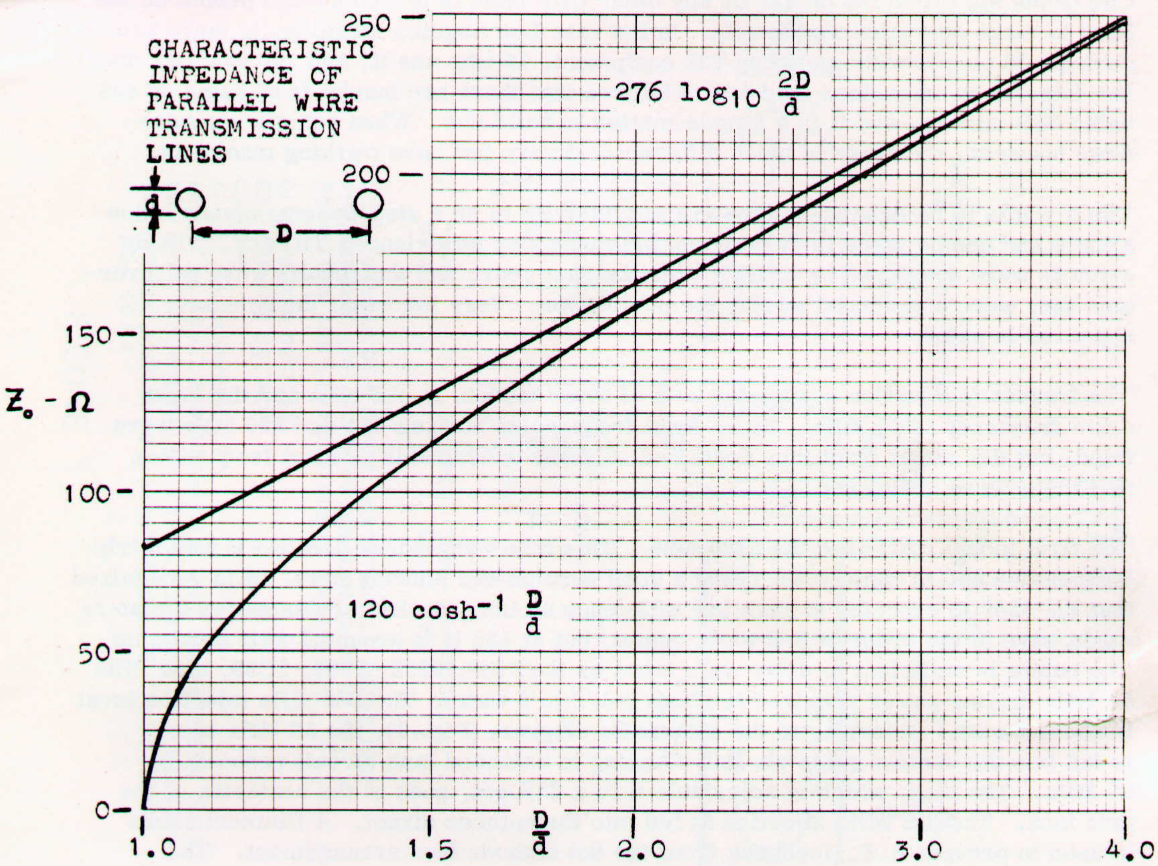
Have you ever tried to match a 100-ohm balanced load to a 50-ohm feed with a quarter-wave open-line Q-section and a 1:1 balun using the handbook formula for characteristic impedance and worked it out--only to find that the center to center spacing was less than the conductor size? The problem is that the commonly used formula has, in fine print, the restriction that the spacing must be very much greater than the conductor size. A few of the better references [1, 2] give a formula without that restriction. It is

$$Z_o = 120 \text{ arccosh } \frac{D}{d}, \quad D = \text{spacing}, \quad d = \text{wire diameter}.$$

Unfortunately in most ham publications the formula looks like hieroglyphics as far as being useful is concerned. The IT&T Handbook, as it is known, not only has tables of cosh x, but also has curves for the characteristic impedance of open wire transmission lines plotted.[3] For Impedance values less than 250 ohms, both curves (the right and the wrong) are plotted in the figure. As larger impedance values are checked, the curves continue to come together. For Z_o= 225 ohms, the error in using the simpler formula is about 1%, but for lower impedances, the error rapidly becomes greater. Note that the simplified formula indicates that the lowest possible impedance for open wire line would be 83.1 ohms but this would be with the two conductors shorted together for their entire length. In the table below are listed the exact D/d ratios for common impedances. These and the curve both assume perfect air insulation with no supports.

Z_o	D/d
0	1.000
25	1.0216
37.5	1.0492
50	1.0882
70.7	1.1784
75	1.2017

Z_o	D/d
100	1.3678
141.4	1.7793
150	1.888
200	2.7672
225	3.350



*221 Abraham Drive, Ames, Iowa 50010

[1]International Telephone and Telegraph Corporation, Reference Data for Radio Engineers, fourth edition, 1956.

[2]Jasik, Antennas, McGraw-Hill Book Company, New York.

[3]IT&T, p. 588.

VHFER magazine is published every other month by Parks Electronics Lab., 419 S.W. First, Beaverton, Ore. 97005. Subscription rates: U.S. & possessions, $1.50 per year, Canada & Mexico, $1.75 per year, others $2 per year. Second class postage paid at Beaverton, Oregon 97005.

Solving the inverse cosh formula for spacing for a given impedance gives the result D/d = cosh(Z0/120).

For a long time ARRL publications have neglected that the 276 log 2D/d formula is only valid for D much greater than d where D is the spacing and d is the conductor diameter. As shown in the graph above that formula says when the conductors are ready to touch the minimum impedance is 83 ohms.

The May 2010 QST still showed the wrong conclusions for low impedance on page 61.

The next answer about spacing is that the characteristic impedance (Z_0) is determined by the ratio of spacing to wire size. The $Z_0 = 276 \times \text{Log}_{10}\ (2\times S/d)$, where S is the center to center spacing of the wires and d is the diameter of each (equal sized) conductor. The closest feasible spacing is with the wires not quite touching at which S is just greater than d, with a corresponding Z_0 of just above 83 Ω (note that at that limit the wires are almost shorted). Using the 4 inches (above) and a wire size of #18 AWG (0.04), we get 552 Ω, so the practical range of Z_0 for most amateur applications is between around 100 and 600 Ω.

I have to admit that only about half the professional references mention the lower impedance limit of the simple formula but until March 2017 I had not found any ARRL publications that do. The section on parallel wire impedance in the 2014 ARRL Handbook on page 20-4 finally gets it correct.

The characteristic impedance of an air-insulated parallel-conductor line, neglecting the effect of the insulating spacers, is given by

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \cosh^{-1} \frac{S}{d} \qquad (4)$$

where

Z_0 = characteristic impedance
S = center to center distance between the conductors
d = diameter of conductors in the same units as S

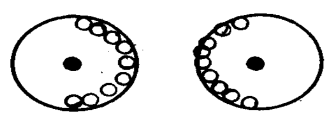
When S >> d, the approximation $Z_0 = 276 \log_{10} (2S/d)$ may be used but for S < 2d gives values that are significantly higher than the correct value, such as is often the case when wires are twisted together to form a transmission line for impedance transformers.

Where ε is the dielectric constant if not air.

The close spaced formula comes from work of Harold Wheeler about 1939 where he found that the close spaced wires caused a concentration of

the charges in the wires where the gap was smallest increasing the capacitance over the forumla that works for wide spacing and that lead to the inverse hyperbolic cosine formula that only gets to zero impedance when the conductors touch.

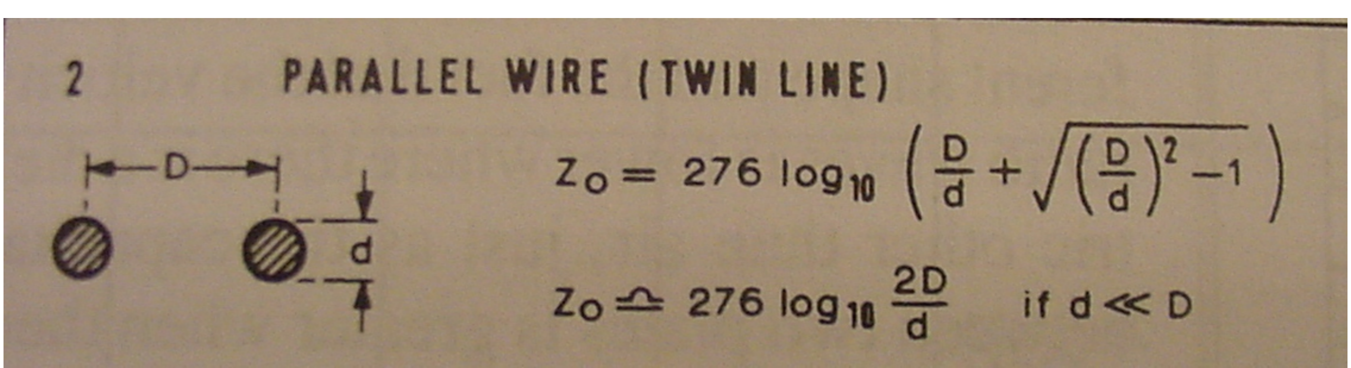
These two crude sketches show extremes in charge distribution, evenly around the conductors for wide spacing and mostly concentrated with close spacing. Not necessarily to scale.



The earliest reference I had with the full formula dates from 1943, an early edition of the Federal Telephone and Telegraph Radio Handbook. It was copied from a Standard Telephone and Telegraph Radio Handbook probably printed about 1942.

ARRL has finally noticed the update after about 72 years.

In April 2017 I found another formula in the 1997 RSGB VHF/UHF


2 PARALLEL WIRE (TWIN LINE)

$$Z_O = 276 \log_{10} \left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1} \right)$$

$$Z_O \triangleq 276 \log_{10} \frac{2D}{d} \quad \text{if } d \ll D$$

Handbook.
This is harder to solve for D/d but gives similar results to the arccosh formula. I ran a spread sheet comparison. The differences are less than 0.1% smaller than most of us would be varying the construction of such a line.

I don't know how the arccosh formula was created, possibly it was fitted to a curve from lab measurements so the RSGB formula is probably as good a fit.

Impedance	COSH spacing	SQRT Impedance
0	1.0000	0.00000
25	1.0218	24.9719
37.5	1.0492	37.4579
50	1.0881	49.9439
70.7	1.1786	70.6206
75	1.2018	74.9158
100	1.3678	99.8877
141.4	1.7784	141.2413
150	1.8884	149.8316
200	2.7417	199.7755
225	3.3371	224.7474
250	4.0779	249.7193
275	4.9963	274.6913
300	6.1323	299.6632

In May 2017 I found another formula in Brian C. Wadell's Transmission Line Handbook from 1991 that is more complex than the 120 arccosh formula but gives the same results. It is noted to only be valid for d<<D which is incorrect. Its good down to contact. There is a 2003 edition of that book with tabulated errata and I take that validity note is one of the errors found later but I've not found a 2003 edition to read the errata.

73, KØCQ