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Bandwidth

This activity started from the need to determine quantitatively what the effective bandwidth of a series of cascaded stages would be when tuned to resonance. Assumptions that would be applicable in a low frequency environment have been made to simplify the calculation but these would likely not affect the overall result. The method shown deals with a series of stages tuned at the same center frequency and not stagger tuned.

Starting from the case of a single stage band-limited by two tuned circuits at the input and output respectively that are tuned to resonance with the assumption that the gain stage is unilateral and has a gain-bandwidth product far greater than the circuits under consideration, the tuned circuits may be converted to RC form instead of RLC. This is allowable due to the fact that $X_l = X_c$ at resonance and will be approximately correct "near" resonance. The circuit is now in the form of two cascaded RC networks with an isolating gain stage inserted between them.

This form may be written as follows:

For a single RC,

$V_o = I_i\,(R + 1/jwC)$

Where $I_i = V_i\,/(R+1/jwC)$

For a dual RC neglecting the gain of the stage,

$V_o = V_i\,((1/jwC/(R+1/jwC)) + (1/jwC/(R+1/jwC)))$

$V_o = V_i\quad(1/(1-R^2w^2C^2 + j2RwC))$

Then, $V_o/V_i = [1-R^2 w^2 C^2] + 2j[RwC]$, rationalizing

$1+ 2R^2 w^2 C^2 + (RwC)^4 = [1+ (RwC)^2]^2$

As an example, determine the 0.1 dB bandwidth for two tuned circuits as a percentage of the 3 dB bandwidth:

For 0.1dB = $10^{.01}$ each tuned circuit must fall off by $10^{.005}$

$(1-R^2 w^2 C^2)^2 + (2RwC)^2 = 10^{.01}$

$1 + 2R^2 w^2 C^2 + (RwC)^4 + 4R^2 w^2 C^2 = 10^{.01}$

$(RwC)^2 = 10^{.005} - 1$
$\sqrt{}RwC = \sqrt{10^{.005} - 1} = .1076$

The above result says that the 0.05 dB bandwidth of each tuned cascade stage is 10.76% of the 3 dB bandwidth or knowing the 0.05 dB bandwidth of each tuned circuit allows the calculation of the required 3 dB bandwidth of each tuned circuit.

In general:

$(RwC)^2 = (10^{((1/10\,dB)/\eta)} - 1)$

Where η = # of synchronously tuned circuits
dB = Desired BW measure in dB